Mathematical Programming Methods for Pressure Management in Water Distribution Systems

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Pressure Management in WDS

Pressure Management has become ubiquitus. Different methods and technologies have been applied in order to keep it close to the minimum allowed by standard regulation.

1. DMAs implementation: improved leakage management but REDUCED redundancy in network connectivity.
2. Water networks with adaptive reconfigurable topology.

To benefit from these advanced control schemes, the retrofit of existing networks requires the solution of both design and operational optimization problems.

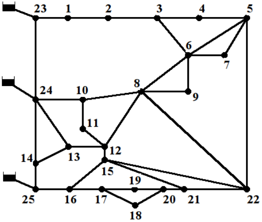
Pressure Management in WDS

As a standard in Water Distribution Network management we consider two types of pressure controllers:

- Pressure Reducing Valves;
- Boundary valves: allow a range of pressure differential along the pipe, as the setting varies from open to close.

We study the problem of simultaneously optimizing both the location of the actuator and the optimal pressure settings.

Optimal valve placement and operation



(a)	(b)
22 junctions	$106,804$ junctions
3 water sources	32 water sources
37 pipes	$87,912$ pipes

Formulation of the mathematical program

We address the minimization of average zone pressure through the placement of n_v control valves.

$$\begin{aligned} \min_{x \in \mathbb{R}^N} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \leq 0, \quad \forall i \in I, \\ & h_i(x) = 0, \quad \forall i \in E, \\ & x_j \in \{0,1\}, \quad \forall j \in B. \end{aligned}$$

The resulting optimization problem is a sparse, non-convex Mixed Integer Nonlinear Program (MINLP).

Reformulation as MPCC

The MINLP can be equivantely reformulated as a Mathematical Program with Complementarity Constraints (MPCC):

$$\begin{aligned} \min_{x \in \mathbb{R}^N} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \leq 0, \quad \forall i \in I, \\ & h_i(x) = 0, \quad \forall i \in E, \\ & 0 \leq x_j \perp 1 - x_j \geq 0, \quad \forall j \in B. \end{aligned}$$

MPCCs have a special structure which violates generic constraints qualifications, causing severe convergence issues to standard solvers.

Penalty Method

Let $\Psi(x) = \sum_{j \in B} x_j(1 - x_j)$. For $\rho > 0$ fixed, consider the nonlinear program PEN(ρ):

$$\begin{aligned} \min_{x \in \mathbb{R}^N} \; & f(x) + \rho\Psi(x) \\ \text{subject to: } & g_i(x) \leq 0, \quad \forall i \in I, \\ & h_i(x) = 0, \quad \forall i \in E, \\ & 0 \leq x_j \leq 1, \quad \forall j \in B. \end{aligned}$$

Relaxation Method

For $t > 0$, consider the nonlinear program REL(t):

$$\begin{aligned} \min_{x \in \mathbb{R}^N} \; & f(x) \\ \text{subject to: } & g_i(x) \leq 0, \quad \forall i \in I, \\ & h_i(x) = 0, \quad \forall i \in E, \\ & 0 \leq x_j \leq 1, \quad \forall j \in B, \\ & \sum_{j \in B} x_j(1 - x_j) \leq t. \end{aligned}$$

Advantages and disadvantages

$\surd$ The orginal difficult MINLP is converted into a series of standard nonlinear programs.
$\surd$ Standard constraints qualifications are satisfied by PEN(ρ) and REL(t) .
$\surd$ PEN(ρ) and REL(t) have a sparse structure.

$\times$ The sequence of penalization and relaxation parameters are not known a priori.
$\times$ When the problem is non-convex only local optimality of the solution is guaranteed.

Case study: 25 nodes network

The selected benchmarking network:

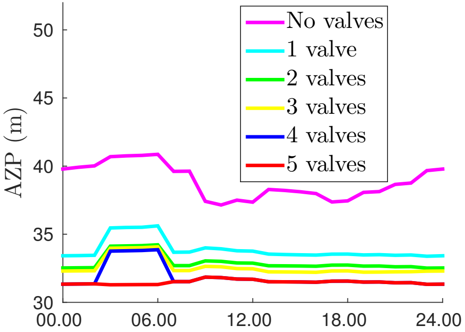
- $\mathbf{22}$ nodes,
- $\mathbf{37}$ pipes
- $\mathbf{3}$ reservoirs.

We divide network's daily operation into $\mathbf{24}$ time steps. The resulting optimization problem is a sparse, non-convex MINLP with $\mathbf{2378}$ variables and $\mathbf{8800}$ constraints.

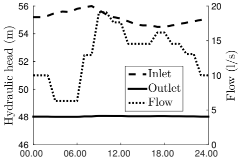
- We compare the reformulation approaches with the MINLP solver BONMIN (v.1.8.1).
- The NLP subproblems within penalty and relaxation methods are solved using the interior point solver for large scale nonlinear optimization IPOPT (v3.11.8).

Solution with BONMIN

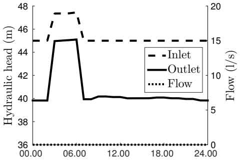
Computational time ranging from $76s$ for the optimization of 1 valve to ~ 32 minutes for 5 valves.



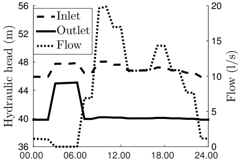
Control profiles for 4 optimized valves



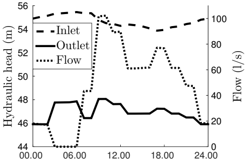
$(23, 1)$



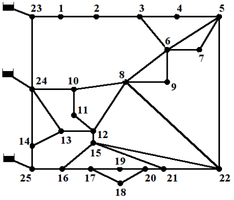
$(22, 15)$



$(12, 15)$



$(13, 12)$



Reformulation approaches

- The relaxation method converges to the best optimal solutions in most instances.
- Both the penalty and relaxation methods converge to the same optimal solutions as Bonmin, or to slightly sub-optimal configurations.
- The computational time is significantly reduced:
  - Penalty method between 100 and 200 seconds.
  - Relaxation method between 20 and 60 seconds.

Conclusions

1. The presented mathematical formulation is flexible to incorporate different physical actuators, such as PRVs and BVs.
2. The relaxation approach is shown to have superior performance both in quality of the solutions and CPU time and can be succesfully applied to design problems for water distribution networks.
3. The relaxed problems have sparse nonlinear structures and so can be solved using tailored techniques for sparse nonlinear programs, offering a scalable approach for large scale WDNs.
4. The presented study demonstrates that the mathematical optimization framework can provide effective tools to support design and operation of WDNs with adaptive network topology.

Thank you!

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For further information

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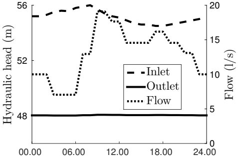
Control profiles for 5 optimized valves



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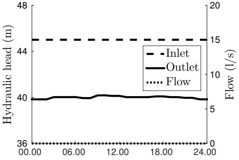

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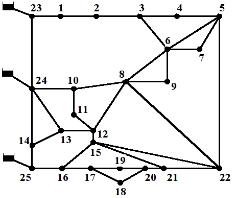

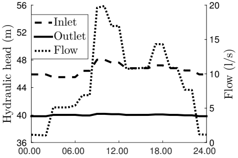

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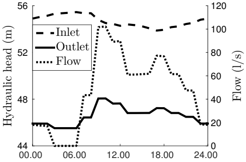

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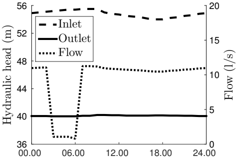

(25,16)

$$\text{Minimize} \quad \sum_{k=1}^{n_l} \frac{1}{W} \sum_{i=1}^{n_n} w_i p_i^k$$

$$\begin{aligned}
\text{subject to:} \quad & A_{12}^T q^k - d^k = 0, \quad \forall k = 1, ..., n_l, \\
& -S(q^k)\big(-A_{12}p^k - A_{12}e - A_{10}h_0^k - h_f(q^k)\big) \leq 0, \ \forall k = 1, ..., n_l, \\
& -A_{12}p^k - A_{12}e - A_{10}h_0^k - h_f(q^k) - M^k v \leq 0, \ \forall k = 1, ..., n_l, \\
& v_j + v_{n_p+j} \leq 1, \quad \forall j = 1, ..., n_p, \\
& \sum_{j=1}^{2n_p} v_j = n_v, \\
& p_{min}^k \leq p^k \leq p_{max}^k, \quad \forall k = 1, ..., n_l, \\
& 0 \leq q_j^k \leq \frac{\pi D_j^2}{4}, \quad \forall j = 1, ..., 2n_p, \ \forall k = 1, ..., n_l, \\
& v \in \{0, 1\}^{2n_p}.
\end{aligned}$$