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Tight convex relaxations for optimal design and control problems in water supply networks

Filippo Pecci[a], Edo Abraham[b], and Ivan Stoianov[a]

[a]Imperial College London

[b]TU Delft

InfraSense Labs
infrasense.net

Imperial College London

Common wisdom

- Optimisation problems in WSNs are non-convex.
- Gradient based optimization method converge to locally optimal solutions.
- In practice, heuristics can provide “near-optimal” solutions.
- Computing global optimality guarantees is impractical.

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This is not true.

Summary

- Design and control problems in water networks
- Global optimality bounds
- Polyhedral relaxations
- Numerical examples

Optimisation problems in WSNs

- **Optimal pipe diameter sizing**

Raghunathan, A.U. (2013) Global Optimization of Nonlinear Network Design. SIAM Journal on Optimization. 23 (1), 268–295.

- **Optimal valve placement and operation**

Pecci, F., Abraham, E., & Stoianov, I. (2018) Global optimality bounds for the placement of control valves in water supply networks. Optimization and Engineering.

- **Optimal pressure control**

Wright, R., Abraham, E., Parpas, P., & Stoianov, I. (2015) Control of water distribution networks with dynamic DMA topology using strictly feasible sequential convex programming. Water Resources Research. 51 (12), 9925–9941.

- **Optimal pump scheduling**

Menke, R., Abraham, E., Parpas, P., & Stoianov, I. (2015) Exploring Optimal Pump Scheduling in Water Distribution Networks with Branch and Bound. Water Resources Management. 30 (14), 5333–5349.

Optimisation problems in WSNs

- Continuous variables can represent flows, hydraulic heads, control inputs

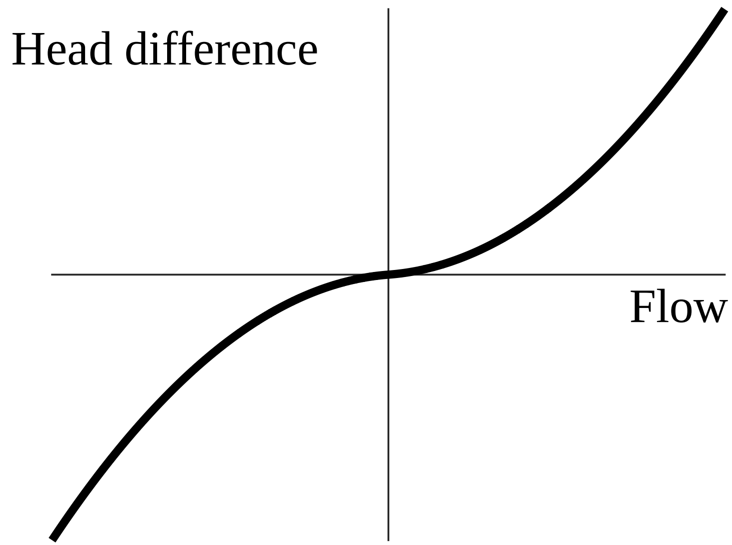
- Discrete variables can represent diameter sizes, valve locations, or pump's status

$$\begin{aligned} \text{minimize} \quad & f(x, z) \\ \text{subject to} \quad & g(x) = 0 \\ & (x, z) \in \mathcal{C} \\ & z \in \mathbb{Z} \end{aligned}$$

- $f(\cdot)$ is a convex objective function.
- $\mathcal{C}$ is a convex set.
- $g(\cdot)$ is a non-linear function.

Non-convex constraints

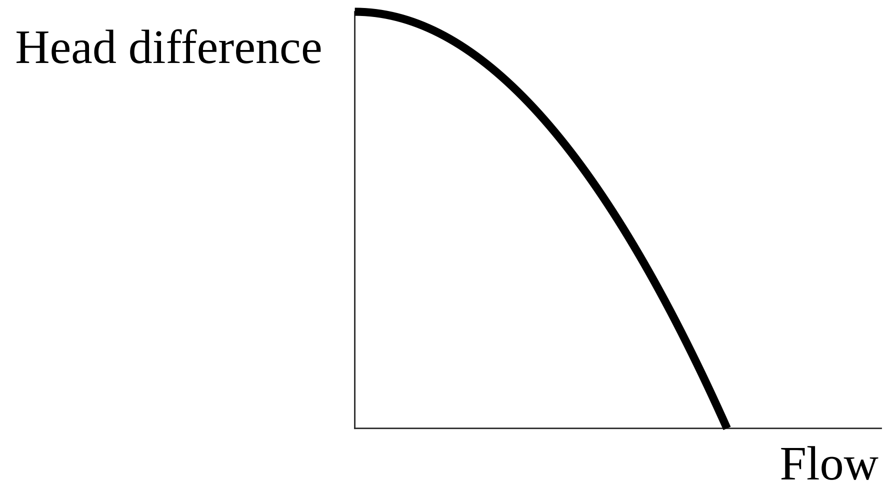
$$g_i(x) = 0$$



- Represent the relation between head difference and flow across a pipe or valve

Non-convex constraints

$$g_i(x) = 0$$



- Represent the relation between head difference and flow across a pump

Non-convex MINLP

$$\begin{aligned} \text{minimize} \quad & f(x,z) \\ \text{subject to} \quad & g(x) = 0 \\ & (x,z) \in \mathcal{C} \\ & z \in \mathbb{Z} \end{aligned}$$

Computing the optimal value p^* is NP-hard.

Non-convex MINLP

$$\begin{aligned} \text{minimize} \quad & f(x, z) \\ \text{subject to} \quad & g(x) = 0 \\ & (x, z) \in \mathcal{C} \\ & z \in \mathbb{Z} \end{aligned}$$

Computing the optimal value p^* is NP-hard.

Can we compute a good quality feasible solution with a certified sub-optimality bound?

Solution method

Aim: Compute a feasible solution with a certified bound to the level sub-optimality

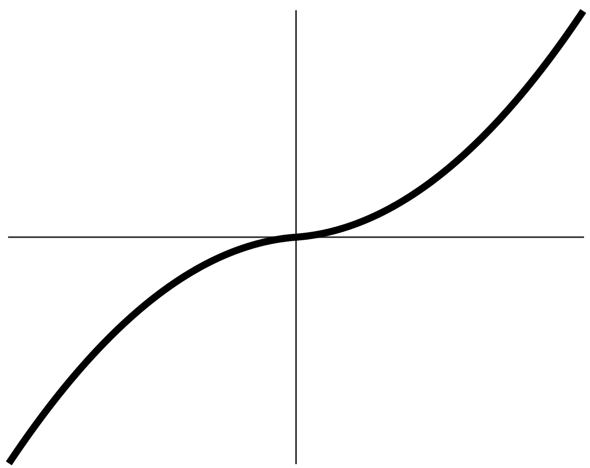
Ingredients:

- A method to compute a lower bound to the optimal value of the non-convex MINLP.
- A method to compute a feasible solution, providing an upper bound to the optimal value of the non-convex MINLP.

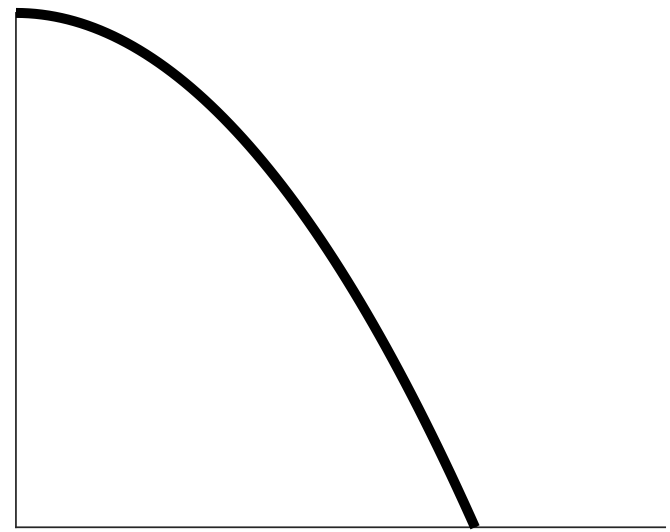
$$\mathrm{LB} \leq p^* \leq \mathrm{UB}$$

Polyhedral relaxations

$$g(x) = 0$$



Pipe/Valve



Pump

Polyhedral relaxations

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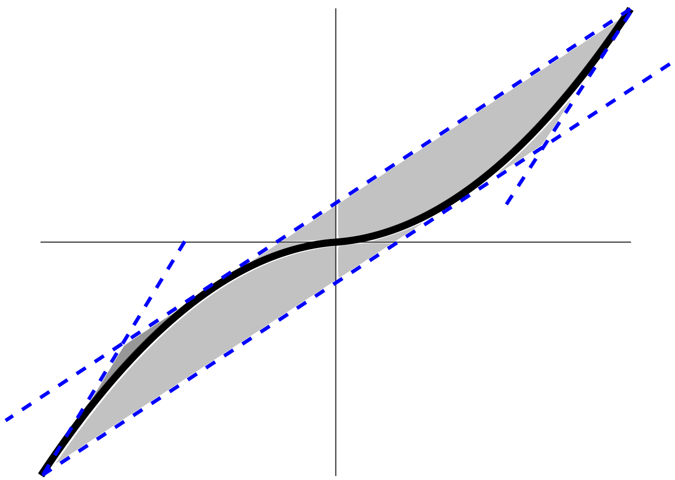
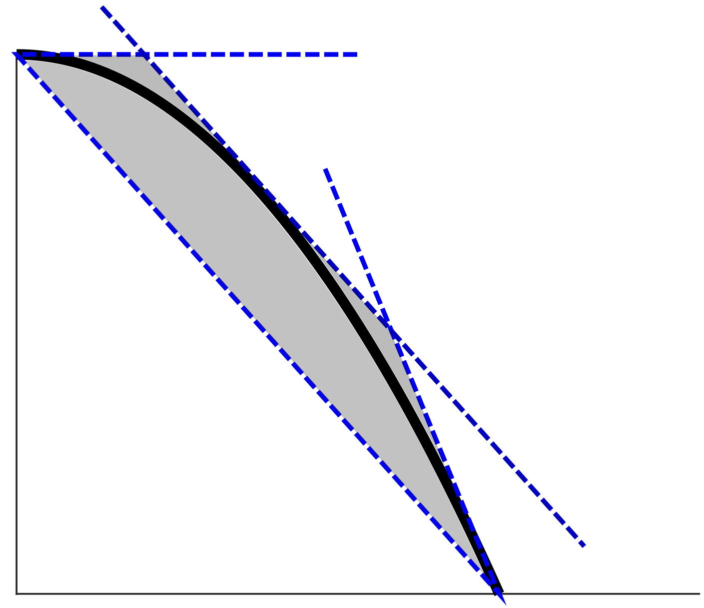
$$Ax \leq b$$



Pipe/Valve



Pump

Lower bounding

$$\begin{array}{ll} \text{minimize} & f(x, z) \\ \text{subject to} & Ax \leq b \\ & (x, z) \in \mathcal{C} \\ & z \in \mathbb{Z} \end{array}$$

It's a convex MIP relaxation!

- Solve the convex MIP relaxation.
- The optimal value provides a lower bound to the optimal value of the original problem:

$$\text{LB} \leq p^*$$

Upper bounding

- Fix the integer variables to the values computed solving the convex MIP relaxation:

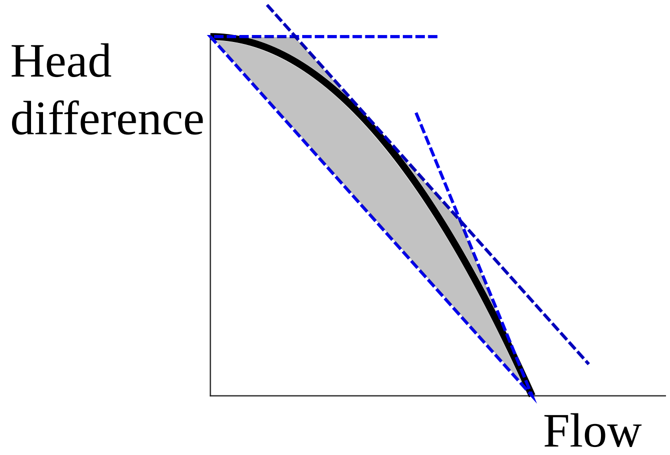
$$\begin{aligned} \text{minimize} \quad & f(x, \hat{z}) \\ \text{subject to} \quad & g(x) = 0 \\ & (x, \hat{z}) \in \mathcal{C} \end{aligned}$$

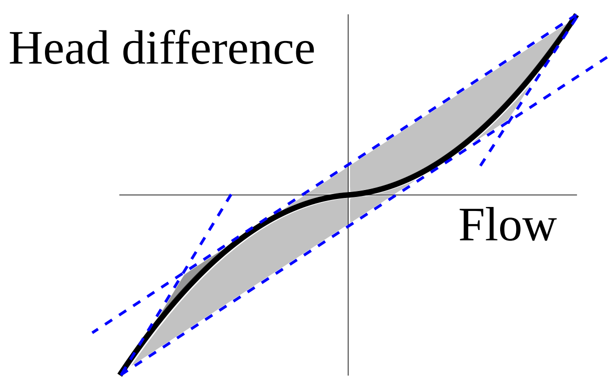
- Solve the resulting non-convex continuous optimization problem using a gradient based method.
- The computed solution provides an upper bound to the optimal value of the original non-convex MINLP:

$$p^* \leq \mathrm{UB}$$

Bound tightening

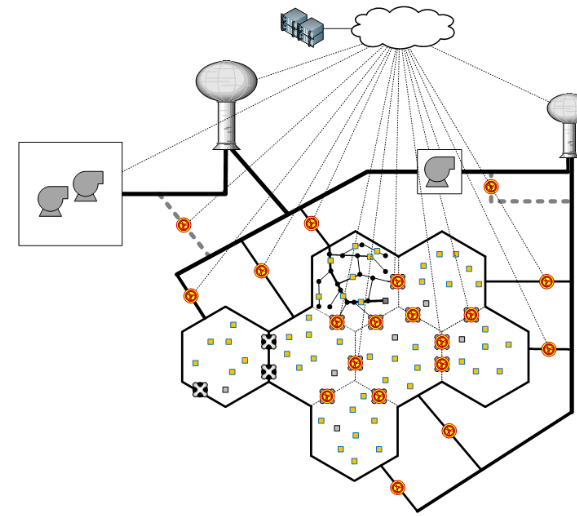
- To improve the computed lower bounds, we tighten the polyhedral relaxations.
- This is done by tightening upper and lower bounds on the flow variables.
- The iterative procedure stops when no more progress is made.
- Details: Pecci, F., Abraham, E., & Stoianov, I. (2018) Global optimality bounds for the placement of control valves in water supply networks. Optimization and Engineering.





Design for Control of WSNs

- **Aim**: minimize Average Zone Pressure (AZP)
- Simultaneously optimise placement and operation of pressure control valves



Pecci, F., Abraham, E., & Stoianov, I. (2018) Global optimality bounds for the placement of control valves in water supply networks. Optimization and Engineering.

Design for Control of WSNs

- Continuous variables
  - Node hydraulic heads
  - Pipe flow rates
  - Pressure control valve settings
- Discrete variables
  - Binary variables used to model the placement of valves
- Non-convex constraints
  - Frictional head losses

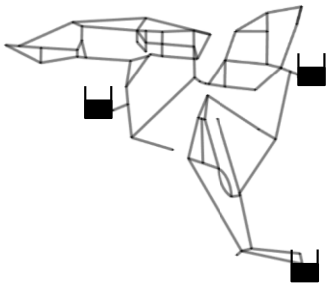
Non-convex Mixed Integer Nonlinear Program (MINLP)

Case studies

Optimal placement and control of 1 to 5 valves in

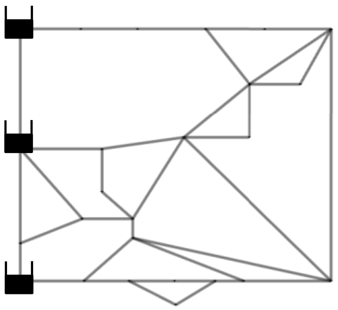
PescaraNet

- 365 continuous variables
- 198 binary variables
- 1591 linear constraints
- 99 non-convex terms



Net25

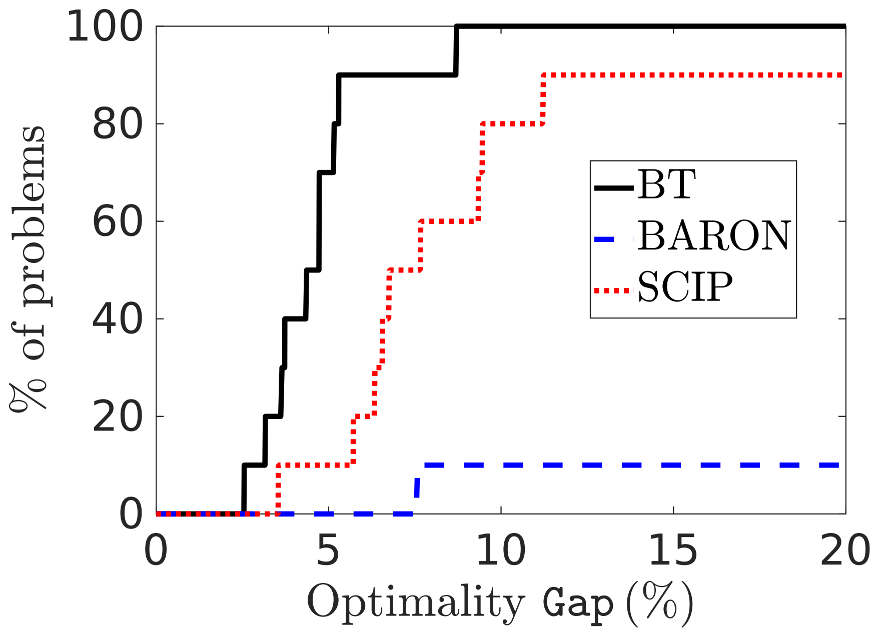
- 3192 continuous variables
- 74 binary variables
- 9762 linear constraints
- 88 non-convex terms



Numerical results

Comparison with solvers BARON (V18.8.23) and SCIP (v3.2.1)

Max Cpu time = 7200 s



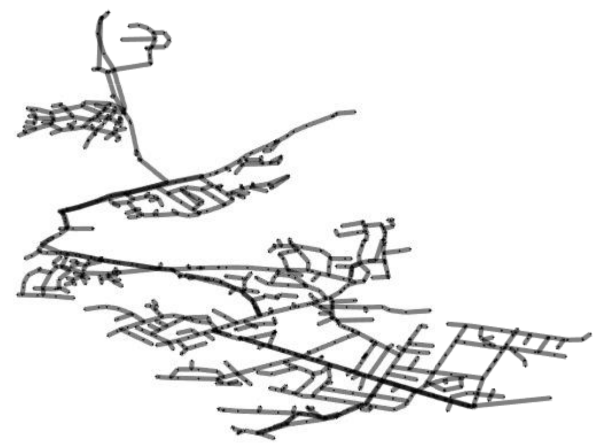
Average CPU times:

- Bound-Tightening algorihtm: 102 s
- BARON: 7200 s
- SCIP: 7200s

Large operational water network

BLWFnet

- 28251 continuous variables
- 2620 binary variables
- 96599 linear constraints
- 7107 non-convex terms



Numerical results

Max Cpu time = 86400 s (1 day)

No. of valves	CPU time (s)	LB	UB	Gap
1	3745	41.73	47.41	13.6 %
2	4803	35.19	39.31	11.6 %
3	40350	32.44	36.19	11.5 %

Bounds on optimality gap comparable to the level of uncertainty experienced within hydraulic models of operational water networks!

Conclusions

- Optimisation problems in WSNs are non-convex, **but the non-convexities are mild.**
- Using polyhedral relaxations, we can build convex relaxations of the original non-convex problems.
- We implement a bound tightening method to improve the lower bounds computed solving the convex relaxations.
- The proposed method yields good quality feasible solutions, with a certified bound on the level of sub-optimality.
- Our simple approach outperforms state-of-the-art global optimisation solvers, for the considered case studies.

Thank you!

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For further information:
f.pecci14@imperial.ac.uk

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InfraSense Labs
infrasense.net