Optimal Design-for-Control of Chlorine Booster Systems in Water Networks via Convex Optimization

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Outline

- Background and motivation
- Problem formulation
- Convex optimization heuristic
- Numerical experiments

Challenges in Water Supply

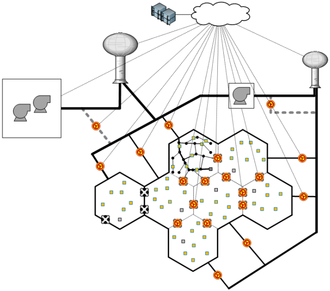
- Growing water demand
- Climate change (drought and flooding)
  - Water supply outages and infrastructure deterioration
- Ageing infrastructure
  - UK urban water infrastructure $>$ 100 years old
- Water quality management to protect public health and provide assurance to customers

Water quality management

- Disinfectant (e.g chlorine) residuals are critical control variables to preserve water quality and eliminate the risks of contamination
- Water utilities aim to maintain optimal target chlorine concentrations
  - Prevent microbial contamination, without affecting water taste and odour, or causing growth of disinfectant by-products
- Optimized chlorine residuals should also avoid spatial and temporal variations, which are perceived as water quality problems by customers

Chlorine booster stations

- Because chlorine is reactive, it is depleted over time as it travels across the pipe networks, causing a reduction in the ability to prevent microbial contamination
- Optimal chlorine dosage is achieved through the operation of chlorine booster stations, which re-apply disinfectant at selected locations within the network



Design-for-control of chlorine booster systems

- We investigate solution methods for the problem of optimal placement and operation of chlorine boosters
- Near real-time control requires the formulation and implementation of appropriate control schemes, e.g. feedback control
- In contrast, we consider a design-for-control problem, where device locations and settings are jointly optimized over a given time interval

Water quality modeling

- A water network with n_n demand nodes, n_0 water sources, and n_p links is modeled as an un-directed graph with $n_n + n_0$ vertices and n_p edges
- As common in literature on water quality optimization, our formulation considers network hydraulic behavior as known and fixed
- The transport of chlorine residuals through each link is governed by a linear PDE for advective transport, which we discretize using a three-point implicit upwind scheme
- Let n_t be the number of discretize time steps

Problem formulation

- Continuous variables
  - concentrations at nodes $c \in \mathbb{R}^{n_t(n_n+n_0)}$
  - concentrations in pipes $p \in \mathbb{R}^{3n_t n_p}$
  - additional concentrations introduced by boosters $u \in \mathbb{R}^{n_t n_n}$
- Binary variables $z \in \{0,1\}^{n_n}$ modeling placement of boosters at nodes
- Given a vector of target concentrations $\bar{c} \in \mathbb{R}^{n_n}$, our objective is to minimize the cumulative target deviation:

$$f(c) = \sum_{i=1}^{n_n} \sum_{k=1}^{n_t} d_{i,k}(c_{i,k} - \bar{c}_i)^2, \tag{1}$$

where $d_{i,k}$ is the water demand of node i and time k

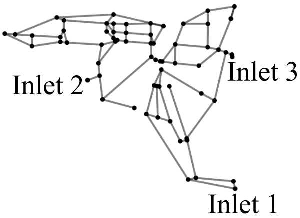
- Our design-for-control problem is formulated as a Mixed Integer Quadratic Program:

$$
\begin{aligned}
\text{minimize} \quad & f(c) \\
\text{subject to} \quad & Ac + Bp + Eu = r \\
& u - Mz \leq 0 \\
& \mathbf{1}^T z = n_b \\
& 0 \leq c \leq c^{\max} \\
& 0 \leq p \leq p^{\max} \\
& 0 \leq u \leq c^{\max} \\
& z \in \{0, 1\}^{n_n}.
\end{aligned} \tag{2}
$$

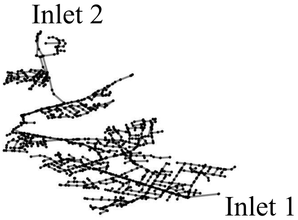
Implementation of state-of-the-art MIP solvers

- Previous literature has formulated and solved the problem of optimal placement of chlorine boosters as a mixed integer program (MIP)
- However, these studies have considered small and medium size water network models with less than a thousand nodes
- We show that state-of-the-art MIP solvers can fail to compute a feasible solution for the considered problem when larger water networks are considered

Case studies



(a) Pescara network model.



(b) BWFLnet network model.

	`Pescara`	`BWFLnet`
n_p	98	2281
n_n	67	2221
n_0	3	2
# Cont. var.	10,344	270,888
# Bin. var.	67	2,221

- We formulate the design-for-control problem for $n_b = 0, \ldots, 10$
- Target concentration at demand nodes is set to 1 mg/l,
- Maximum concentration at demand nodes and water sources are not allowed to exceed 2 mg/l

Results with MIP solvers

- Solvers CPLEX and GUROBI are implemented to directly compute globally optimal solutions for our MIQP
- We set a time limit of two hours. If a solver has not converged to a globally optimal solution within the time limit, we return the best feasible solution at that point.
- When considering `Pescara`, the solvers have converged to the same solutions for all problem instances, with global optimality guarantees obtained only for $n_b \leq 6$.
- In the case of `BWFLnet`, CPLEX and GUROBI did not find any feasible solution within the time limit of two hours for all $n_b \geq 1$.

Table: Optimal values and computational times reported for CPLEX and GUROBI in Pescara. We denote with a * experiments where the solvers did not converge to a globally optimal solution.

n_b	CPLEX		GUROBI	
	Obj. Value	Time (s)	Obj. Value	Time (s)
0	32.86	0.21	32.86	0.09
1	26.57	4.83	26.57	8.41
2	23.17	12.90	23.06	19.54
3	23.06	60.57	19.77	125.67
4	17.32	1426.02	17.32	430.72
5	15.46	2531.53	15.46	1362.69
6	13.92	5066.02	13.92	4619.59
7	12.62*	7200	12.62*	7200
8	11.25*	7200	11.25*	7200
9	10.02*	7200	10.02*	7200
10	8.85*	7200	8.85*	7200

A convex heuristic for large water networks

- State-of-the-art MIP solvers can fail to compute feasible solutions for the considered MIQPs when large water networks are considered
- We investigate a heuristic algorithm based on convex optimization to compute good quality feasible solutions
- The convex heuristic enables the optimal placement and operation of chlorine boosters in large networks like `BWFLnet`

Step 1. Solve a convex relaxation

- Let z^* be a solution of the following continuous convex relaxation:

$$
\begin{array}{ll}
\text{minimize} & f(c) \\
\text{subject to} & Ac + Bp + Eu = r \\
& u - Mz \leq 0 \\
& \mathbf{1}^T z = n_b \\
& 0 \leq c \leq c^{\max} \\
& 0 \leq p \leq p^{\max} \\
& 0 \leq u \leq c^{\max} \\
& z \in [0,1]^{n_n}.
\end{array}
\tag{3}
$$

- We denote by L the corresponding optimal value, which is a lower bound for the original MIQP.

Step 2. Round

- Let $Z \subseteq \{1, \ldots, n_n\}$ be the index set corresponding to the n_b largest components in z^*
- Define $\hat{z} \in \{0,1\}^{n_n}$ as $\hat{z}_i = 1$ if $i \in Z$, and $\hat{z}_i = 0$ otherwise.
- Compute an upper bound $U(\hat{z})$ by solving the quadratic program (QP):

$$
\begin{array}{ll}
\text{minimize} & f(c) \\
\text{subject to} & Ac + Bp + Eu = r \\
& u \leq M\hat{z} \\
& 0 \leq c \leq c^{\max} \\
& 0 \leq p \leq p^{\max} \\
& 0 \leq u \leq c^{\max}
\end{array}
\tag{4}
$$

Step 3. Swap

- The algorithm starts from $\hat{z}$ and checks booster configurations that are obtained swapping one of the n_b locations identified by $\hat{z}$ for one of the locations that were not selected
- Each alternative booster configuration z_{test} is evaluated by solving the corresponding QP and computing $U(z_{\text{test}})$
- If no swap reduces the objective function value, the algorithm terminates.
- If we find a feasible solution with a reduced objective function value, we update $\hat{z}$ and we start a new swapping iteration

- In our implementation, we limit the maximum number of iterations by setting $N_{\mathsf{loc}} = 100 n_n$
- To reduce the number of configurations that are checked by the algorithm, we only considers for swapping booster locations corresponding to indices such that $0.1 \leq z_i^* \leq 0.9$
- The selected booster locations to be removed from $\hat{z}$ are chosen according to their corresponding z^* values in ascending order
- Analogously, the booster locations to be added are selected in descending order

Numerical tests/1

- In the case of `Pescara`, the convex optimization heuristic has converged to feasible solutions that are very close to those of GUROBI and CPLEX.

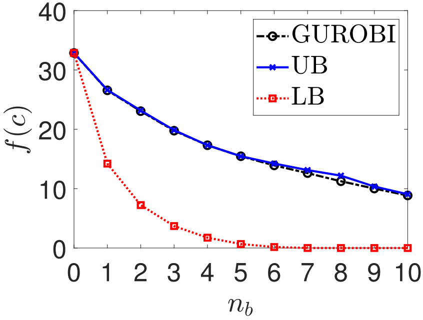


Figure: The computed upper bounds are within 1% of the globally optimal solutions computed by GUROBI for $n_b \leq 6$

- The computational time for the two MIP solvers is one to two orders of magnitude longer than that of the convex heuristic.

Numerical tests/2

- Recall that, in the case of `BWFLnet`, CPLEX and GUROBI did not find any feasible solution within the time limit of two hours for all $n_b \geq 1$.

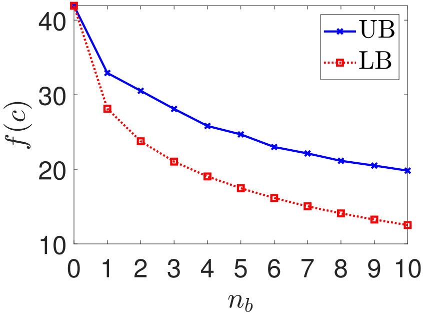


Figure: The computed upper bounds are within 1% of the globally optimal solutions computed by GUROBI for $n_b \leq 6$

- The longest computational time for the convex heuristic in `BWFLnet` is 38 minutes.

Simulate optimized chlorine boosters

- We compare chlorine concentrations obtained solving our MIQP with those simulated by EPANET with the optimized chlorine injections at water sources and optimized booster settings

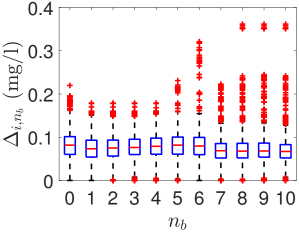


Figure: Statistics of the temporal average of the absolute differences in chlorine concentrations

- The largest errors are experienced at nodes with very low or zero demand

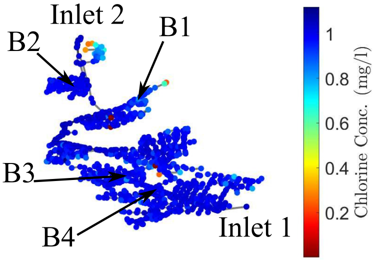


Figure: Distribution of simulated chlorine concentrations at $20:00$ in `BWFLnet`, when four boosters are optimally located and operated.

Conclusions

- The problem of optimal placement and operation of chlorine boosters results in a MIQP, where the number of integer decision variables grows with the size of the considered WDN
- Computing globally optimal solutions requires impractical computational effort when large WDN models are considered
- We have presented a convex heuristic method to generate feasible solutions by solving convex quadratic programs
- The numerical results are promising; the algorithm has resulted in good quality solutions for two case studies, including a large operational water network from the UK

Thank you!

Any question?

I look forward to seeing you in London (and online!)

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