Imperial College London

A branch and bound method for globally optimising valve locations in water distribution networks

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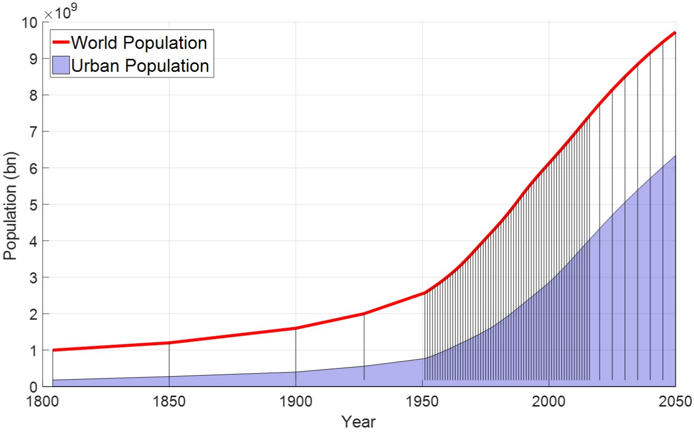
Outline

Challenges in water supply: growing water demand



Challenges in water supply: climate change (drought and flooding)

Water supply outages and infrastructure deterioration

Challenges in water supply: ageing infrastructure

UK urban water infrastructure $>$ a century old (Ofwat)

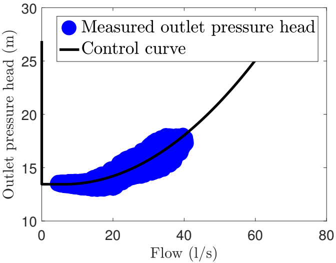
Pressure management in water networks

Hydraulic pressure in pipes is a critical control variable for WSNs:

- Leakage losses
- Pipe bursts frequency

Pressure control valves

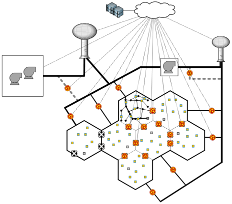
- Control pressure at their outlets
- Advanced forms of flow and pressure modulation



Outline

Optimal placement of control valves in WSNs

- Objective
  - Minimize Average Zone Pressure (AZP)
- Continuous variables
  - Node hydraulic heads
  - Pipe flow rates
- Discrete variables
  - Binary variables used to model the placement of valves
- Nonconvex constraints
  - Frictional energy losses within hydraulic conservation laws



⇒ Nonconvex Mixed Integer Nonlinear Programming (MINLP)

Problem formulation

$$
\begin{aligned}
\text{minimize} \quad & c^T u \\
\text{subject to} \quad & f_i(x_i) - y_i = 0, \quad \forall i = 1, \ldots, m \\
& Ax + By + Cu + Dz \leq d \\
& x \in Q \\
& z_j \in \{0, 1\}, \quad \forall j = 1, \ldots, n
\end{aligned}
\tag{MINLP(Q)}
$$

- $f_i(\cdot)$ is a nonlinear function modelling frictional energy losses
- Q is a rectangle representing variable upper and lower bounds.

Previous approaches

Heuristics:

- Genetic algorithms
- Simulated annealing
- Harmony search
- ...

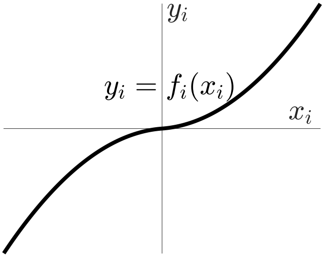
Mathematical optimisation:

- Branch and bound on binary variables
- Penalty method
- Linear approximation

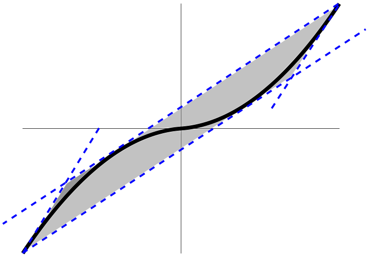
No guarantee on the global optimality of solutions

Outline

Polyehdral relaxation for frictional energy losses constraints



Polyehdral relaxation for frictional energy losses constraints



Defined by

$$R_i x_i + E_i y_i \leq r_i, \quad \forall i = 1, \ldots, m$$

where matrices R_i, E_i, and vector r_i depend on the rectangle Q.

Presolving : domain reduction

idea: Improve the polyhedral relaxation by solving

$$\begin{aligned} \text{minimize/maximize} \quad & x_l \\ \text{subject to} \quad & R_i x_i + E_i y_i \le r_i, \quad \forall i = 1, \ldots, m \\ & Ax + By + Cu + Dz \le d \\ & x \in Q \\ & z_j \in [0,1], \quad \forall j = 1, \ldots, n \end{aligned}$$

for each $l = 1, \ldots, m$.
output: A tightened rectangle Q^{tight}

Branch and bound: basic idea

goal: Find a global optimal solution for MINLP(Q^{tight}) to within some prescribed accuracy ε

- Define a partition $\mathscr{Q}$ of Q^{tight}
- For each $Q' \in \mathscr{Q}$, compute lower and upper bounds on the optimal value of MINLP(Q'):

$$L(Q') \leq y(Q') \leq U(Q'))$$

- If $\min_{Q' \in \mathscr{Q}} U(Q') - \min_{Q' \in \mathscr{Q}} L(Q') < \varepsilon$, quit
- else, refine partition $\mathscr{Q}$ and repeat

Ingredients for branch and bound

The algorithm needs

- methods to compute lower and upper bounds
- strategy to select the next region to split
- rule for choosing how to split

Lower bound

Given $Q' \in \mathscr{Q}$, a lower bound is computed solving

$$\begin{aligned} \text{minimize} \quad & c^T u \\ \text{subject to} \quad & R_i x_i + E_i y_i \leq r_i, \quad \forall i = 1, \ldots, m \\ & Ax + By + Cu + Dz \leq d \\ & x \in Q' \\ & z_j \in \{0, 1\}, \quad \forall j = 1, \ldots, n \end{aligned} \qquad (\text{MILP}(Q'))$$

Let $(\hat{x}, \hat{y}, \hat{u}, \hat{z})$ be the solution of MILP(Q')

Upper bound

Given $Q' \in \mathscr{Q}$, an upper bound is computed solving

$$\begin{aligned}
\text{minimize} \quad & c^T u \\
\text{subject to} \quad & f_i(x_i) - y_i = 0, \quad \forall i = 1, \dots, m \\
& Ax + By + Cu \le d - D\hat{z} \\
& x \in Q'
\end{aligned} \qquad (\text{NLP}(Q'))$$

Ingredients for branch and bound

The algorithm needs

- methods to compute lower and upper bounds ✓
- strategy to select the next region to split
- rule for choosing how to split

Branching strategy

- Select the rectangle $Q^b \in \mathscr{Q}$ with the best lower bound, i.e.

$$L(Q^b) = \min_{Q' \in \mathscr{Q}} L(Q')$$

- Split Q^b at $\hat{x}$, along coordinate k corresponding to the largest error

$$|f_k(\hat{x}_k) - \hat{y}_k| = \max_{i=1,\dots,m} |f_i(\hat{x}_i) - \hat{y}_i|$$

Ingredients for branch and bound

The algorithm needs

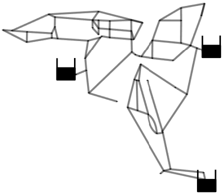
- methods to compute lower and upper bounds ✓
- strategy to select the next region to split ✓
- rule for choosing how to split ✓

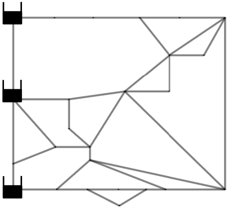
Outline

Case studies

Optimal placement of 1 to 5 pressure control valves in

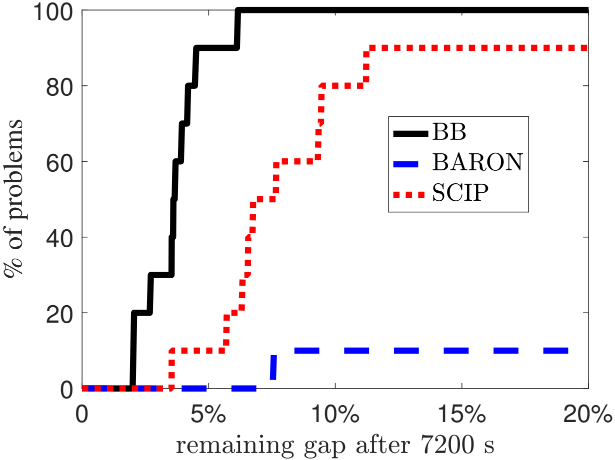
- PescaraNet
  - 365 cont. var.
  - 198 bin. var.
  - 1591 lin. constr.
  - 99 nonconvex terms
- Net25
  - 3192 cont. var.
  - 74 bin. var.
  - 9762 lin. constr.
  - 888 nonconvex terms





Numerical results

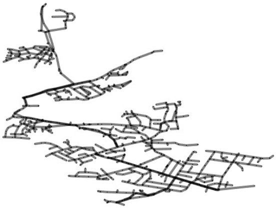
Comparison with solvers BARON (v18.8.23) and SCIP (v3.2.1).



Large operational water network

Optimal placement of 1 to 5 pressure control valves in

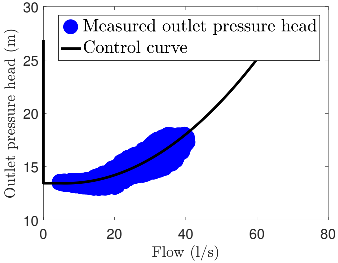
- BWFLnet
  - 28251 cont. var.
  - 2620 bin. var.
  - 96599 lin. constr.
  - 7107 nonconvex terms



Numerical results

n_v	Time (s)	No. Iter.	LB (m)	UB (m)	Gap (%)
1	86400	783	42.48	47.41	11.61
2	86400	29	35.54	39.31	10.62
3	86400	1	32.44	36.19	11.58
4	> 86400	-	-	-	-
5	> 86400	-	-	-	-

Table: Optimization results on BWFLnet ($86400\,s = 1\,$day)



Thank you!

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NEC InfraSense Labs infrasense.net

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