Outer approximation methods for the solution of co-design optimisation problems in water distribution networks

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Water Distribution Networks: complex systems

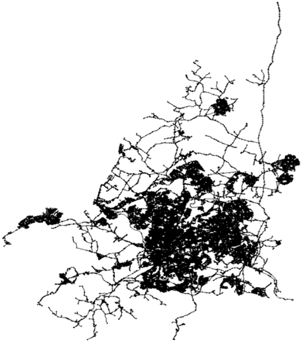


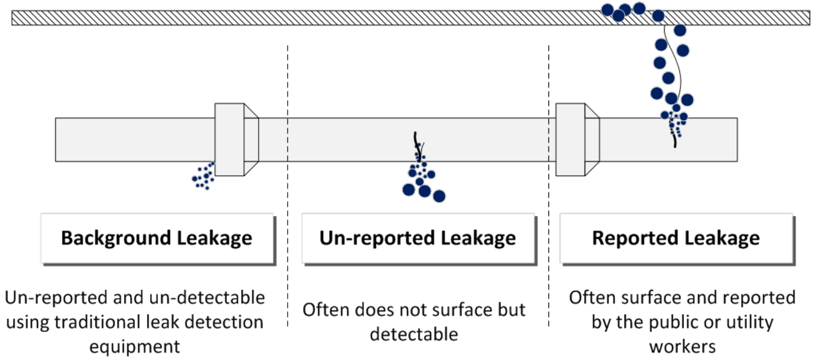
Figure: Network model from UK, 1m POP

Figure: Oxford Street, London (UK), 2013

Improve quality of service

- Reduce leakage
- Reduce risk of pipe failure

Water loss reduction through optimal pressure management



Pressure Management in WDNs

Objectives:

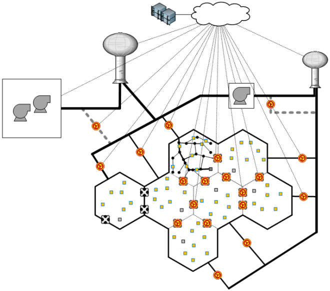
- Minimisation Average Zone Pressure (AZP)
  - reduce pressure-driven leakage
- Minimisation of Pressure Variability
  - fatigue induced pipe failures $\rightarrow$ “calm networks”

Actuators:

- Pressure Reducing Valves (PRVs)
  - reduce pressure at their downstream node

Co-Design Optimisation Problems in WDNs

- Continuous decision variables
  - Flow rates in network pipes
  - Hydraulic pressure heads
- Discrete decision variables
  - Binary variables used to model the placement of valves
- Nonconvex optimisation constraints
  - Hydraulic conservation laws



Mathematical model of a WDN

- A water network with n_0 water sources, n_n nodes and n_p pipes is modeled as a graph with $n_n + n_0$ vertices and n_p links
- We include in the formulation $n_l = 24$ different demand scenarios:
  - $d_i^t \quad \forall i \in \{1, \ldots, n_n\}, \quad \forall t \in \{1, \ldots, n_l\}$
  - $h_{0_i}^t \quad \forall i \in \{1, \ldots, n_0\}, \quad \forall t \in \{1, \ldots, n_l\}$
- Quadratic approximation of frictional head losses:

$$\phi_j(q_j^t) := (a_j|q_j^t| + b_j)q_j^t, \quad \forall j = 1, \ldots, n_p,\ t = 1, \ldots, n_l$$

Optimisation decision variables

- Flow rate across each link q_j^t
- Hydraulic head at each node h_i^t
- Head losses introduced by the valves $\eta^t \in \mathbb{R}^{n_p}$
- Vectors of binary variables $v^+ \in \{0,1\}^{n_p}$ and $v^- \in \{0,1\}^{n_p}$ to model the placement of valves:
  - $v_j^+ = 1 \Leftrightarrow$ valve on link j in the assigned positive flow direction
  - $v_j^- = 1 \Leftrightarrow$ valve on link j in the assigned negative flow direction
  - $v_j^+ = v_j^- = 0 \Leftrightarrow$ no valve is placed on link j
  - $v_j^+ + v_j^- \leq 1$ precludes the placement of two valves on the same physical link

Reformulation of Energy Conservation Laws

Highly nonlinear formulation $\Longrightarrow$ New simplified formulation:

(Eck and Mevissen, 2012; Dai and Li, 2014; Pecci et al, 2016)

$$Q(q^t)(-A_{12}h^t - A_{10}h_0^t - \Phi(q^t)) \geq 0$$

$$-A_{12}h^t - A_{10}h_0^t - \Phi(q^t) - N^t v \leq 0$$

$$0 \leq q^t \leq q^{max}$$

where $v = \begin{bmatrix} v^+ \\ v^- \end{bmatrix}$ and

$q^t \in \mathbb{R}^{2n_p}$

$$\Phi(q^t) + A_{12}h^t + A_{10}h_0^t + \eta^t = 0$$

$$\eta^t - N^{+t}v^+ \leq 0$$

$$-\eta^t - N^{-t}v^- \leq 0$$

$$q^t + Q^{max}v^- \leq q^{max}$$

$$-q^t + Q^{max}v^+ \leq q^{max}$$

- Reduced nonlinearity
- Less nonlinear constraints
- Most variables appear linearly

Overall Problem Formulation

$$\text{Minimise} \quad \frac{1}{n_l W} \sum_{t=1}^{n_l} \sum_{i=1}^{n_n} w_i (h_i^t - e_i) \tag{1}$$

$$\text{subject to} \quad \Phi(q^t) + A_{12} h^t + A_{10} h_0^t + \eta^t = 0, \quad t = 1, \ldots, n_l, \tag{2}$$

$$A_{12}^T q^t - d^t = 0, \quad t = 1, \ldots, n_l, \tag{3}$$

$$\eta^t - N^{+t} v^+ \leq 0, \quad t = 1, \ldots, n_l, \tag{4}$$

$$-\eta^t - N^{-t} v^- \leq 0, \quad t = 1, \ldots, n_l, \tag{5}$$

$$q^t + Q^{max} v^- \leq q^{max}, \quad t = 1, \ldots, n_l, \tag{6}$$

$$-q^t + Q^{max} v^+ \leq q^{max}, \quad t = 1, \ldots, n_l, \tag{7}$$

$$h_{min}^t \leq h^t \leq h_{max}^t, \quad t = 1, \ldots, n_l, \tag{8}$$

$$v^+ + v^- \leq \mathbf{e}, \quad \sum_{k=1}^{n_p} (v_k^+ + v_k^-) = n_v, \quad v^+, v^- \in \{0,1\}^{n_p}. \tag{9}$$

Solution methods

$$
\begin{aligned}
\text{Minimise} \quad & f(x) \\
\text{subject to} \quad & c(x) = 0, \\
& x \in X(v^+, v^-), \\
& (v^+, v^-) \in V
\end{aligned}
$$

We investigate the use of **Outer Approximation (OA)** methods, based on the solution of an alternating sequence of

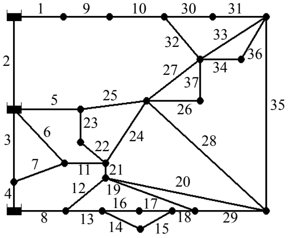
- nonlinear programs (NLP) subproblems
- linear relaxations of the original MINLP

As the problem is nonconvex, such methods are applied as heuristics!

- OA/ER (Kocis and Grossman, 1987)
- AP/OA/ER (Viswanathan and Grossman, 1990)

Case study 1: published benchmarking WDN

- Benchmarking water network used in Eck and Mevissen (2012); Dai and Li (2014); Pecci et al. (2016)
  - $n_n = 22$, $n_p = 37$, $n_0 = 3$, $n_l = 24$
  - No. cont. var. 2304
  - No. bin. var. 74
  - No. lin. const. 5174
  - No. nonlin. const. 888



Case study 1 - Bonmin vs OA

Table: Best known solutions obtained by the solver `Bonmin`

n_v	Link	AZP	CPU time	B-BB iter
1	11	33.63 m	9 s	69
2	11, 1	32.67 m	396 s	3114
3	11, 1, 21	32.16 m	606 s	13667
4	11, 1, 21, 8	31.75 m	878 s	21381
5	11, 1, 21, 8, 20	31.47 m	3306 s	116243

3 orders CPU reduction → scalable approach for larger systems

Table: Local solutions from OA/ER

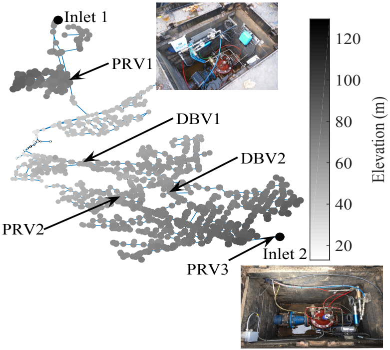
n_v	Link	AZP	CPU time	OA/ER iter
1	11	33.63 m	0.87 s	2
2	11, 1	32.67 m	1.18 s	2
3	11, 1, 5	32.46 m	1.5 s	2
4	11, 1, 5, 21	31.95 m	1.27 s	2
5	11, 1, 5, 21, 8	31.75 m	1.27 s	2

Table: Local solutions from AP/OA/ER

n_v	Link	AZP	CPU time	AP/OA/ER iter
1	11	33.63 m	2.05 s	3
2	11, 1	32.67 m	2.60 s	3
3	11, 1, 5	32.46 m	6.00 s	3
4	11, 1, 5, 21	31.95 m	5.80 s	3
5	11, 1, 5, 21, 20	31.56 m	14.00 s	4

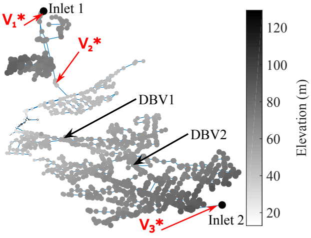
Case study 2: operational network

- Smart Water Network Demonstrator operated by Bristol Water, InfraSense Labs at Imperial College London and Cla-Val.
  - $n_n = 2374$, $n_p = 2434$, $n_0 = 2$, $n_l = 24$
  - No. cont. var. 221808
  - No. bin. var. 4864
  - No. lin. const. 407027
  - No. nonlin. const. 58412

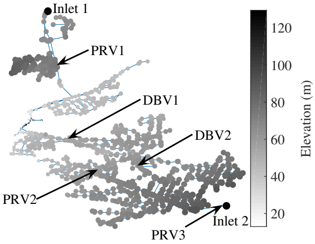


Case study 2 - Results

Solved with OA/ER for the placement of 3 valves in ≈ 2.5 hours



AZP=$36.72\ m$



AZP=$37.02\ m$

Summary and conclusions

- The problem of optimal valve placement and operation in WDNs is a nonconvex MINLP (difficult to solve!).
- New formulation that reduces degree of nonlinearity with respect to previous literature.
- Application of OA algorithms as heuristics for solving this MINLP.
- Promising results on a published benchmarking network model and an operational WDN from the UK.

Thank you!

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