A global optimization framework for resilient water distribution networks

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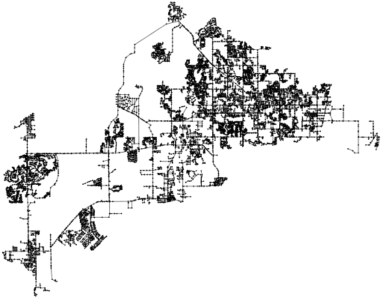
Water Distribution Networks : Complex Systems

WDNs aim to secure supply of potable water:

availability, affordability, and water quality



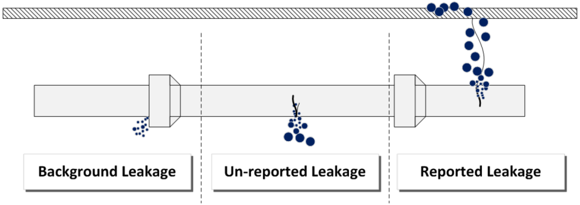
Challenges in Water Supply

- Growing water demand.
- Climate change (drought and flooding).
  - Water supply outages and infrastructure deterioration.
- Ageing infrastructure.
  - UK urban water infrastructure is over 100 years old.

Figure: Po river in June 2022. *S. Levantesi, "Italy must prepare for a future of chronic drought", Nature Italy (2022)*

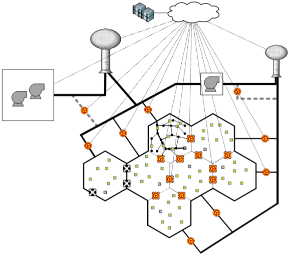
Water Leakage

- Thames Water (which supplies London) loses 24% of the total water supply $\approx$ 250 Olympic swimming pools every day.
- In Italy, ISTAT reports that water leaks correspond to 36% of the total supply.
- Most water leaks are not reported and can not be repaired.



Leakage reduction through pressure control

- Water leakage is a non-decreasing function of network pressure.
- Pressure management is implemented through the operation of automatic control valves.
  - Allow advanced forms of pressure and flow control.
  - Dynamically adapt network topology to minimize pressure and improve resilience.



Optimal design-for-control of WDNs

Joint optimization of locations and operational settings of automatic control valves in WDNs.

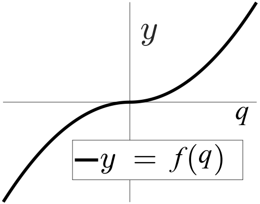
$$
\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{n_t} c^T x_t \\
\text{subject to} \quad & A x_t + f(q_t) = 0, \quad \forall t \in \{1, \dots, n_t\} \\
& B_t x_t + C_t q_t + D_t v \leq e_t, \quad \forall t \in \{1, \dots, n_t\} \\
& x_t \in X_t, q_t \in Q_t \quad \forall t \in \{1, \dots, n_t\} \\
& \sum_{i=1}^{m} v_i = n_v \\
& v \in V \cap \{0, 1\}^m
\end{aligned}
$$

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{n_t} c^T x_t \quad \text{Average Zone Pressure (AZP)} \\
\text{subject to} \quad & A x_t + f(q_t) = 0 \quad \text{Nonlinear potential loss} \\
& B_t x_t + C_t q_t + D_t v \leq e_t, \quad \forall t \in \{1, \ldots, n_t\} \\
& x_t \in X_t, q_t \in Q_t \quad \forall t \in \{1, \ldots, n_t\} \\
& \sum_{i=1}^{m} v_i = n_v \\
& v \in V \cap \{0,1\}^m
\end{aligned}
$$

- Vector $x_t \in \mathbb{R}^p$ includes node hydraulic heads (pressures).
- Vector $q_t \in \mathbb{R}^n$ includes pipe flows.
- Vector $v \in \{0,1\}^m$ models design solutions (e.g. valve locations)
- $X_t \subset \mathbb{R}^p$, $Q_t \subset \mathbb{R}^n$ and $V \subset \mathbb{R}^m$ are polyhedral sets

WDNs are nonlinear network systems

- Flow through the network is driven by a potential.
- The potential loss across a link is a nonlinear function of the flow rate through that link.
- $f_i(q_{it}) = a_i q_{it}|q_{it}| + b_i q_{it}$,
  with $a_i \geq 0$, $b_i \geq 0$



Solution of non-convex MINLPs

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{n_t} c^T x_t \\
\text{subject to} \quad & A x_t + f(q_t) = 0, \quad \forall t \in \{1, \ldots, n_t\} \\
& B_t x_t + C_t q_t + D_t v \leq e_t, \quad \forall t \in \{1, \ldots, n_t\} \\
& x_t \in X_t, q_t \in Q_t \quad \forall t \in \{1, \ldots, n_t\} \\
& \sum_{i=1}^{m} v_i = n_v \\
& v \in V \cap \{0, 1\}^m
\end{aligned} \tag{P}
$$

- Combines complexities in dealing with non-convex constraints with those arising from the presence of discrete decision variables.

State-of-the-art

Off-the-shelf MINLP solvers can fail to compute feasible solutions even for small case studies.

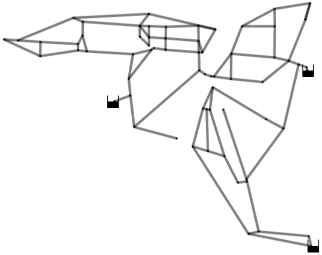


Figure: Benchmarking network of Pescara.

- 70 nodes, 98 pipes, 24 time steps
- 5 problems instances with $n_v \in \{1, \ldots, 5\}$.
- 6312 continuous variables.
- 196 binary variables.
- 2352 non-convex terms.

State-of-the-art

After 6 hours of computations:

- BARON failed to compute a feasible solution for all cases.
- SCIP computed a feasible solution only for $n_v = 1$, with an estimate on the optimality gap of 53%.
- OCTERACT computed feasible solutions only for $n_v = 1, 2, 3$, and the smallest estimate on the optimality gap was equal to 33%.

Analogous performances were reported in previous works:

- F. Pecci, I. Stoianov, and A. Ostfeld. "Relax-tighten-round algorithm for optimal placement and control of valves and chlorine boosters in water networks". In: European Journal of Operational Research 295.2 (2021), pp. 690–698. doi: doi:10.1016/j.ejor.2021.03.004.
- F. Pecci, E. Abraham, and I. Stoianov. "Global optimality bounds for the placement of control valves in water supply networks". In: Optimization and Engineering 20.2 (2019), pp. 457–495. doi:doi:10.1007/s11081-018-9412-7.

A tailored global optimization framework

- We implement a multi-tree method inspired by non-convex generalised benders decomposition schemes.
- These methods have been shown to be effective in solving MINLPs in the framework of power grids.
  - J. Liu, C. D. Laird, J. K. Scott, J. P. Watson, and A. Castillo, “Global Solution Strategies for the Network-Constrained Unit Commitment Problem with AC Transmission Constraints,” IEEE Trans. Power Syst., vol. 34, no. 2, pp. 1139–1150, 2019, doi: doi:10.1109/TPWRS.2018.2876127.
  - Frank and S. Rebennack, “Optimal design of mixed AC-DC distribution systems for commercial buildings: A nonconvex generalized benders decomposition approach,” Eur. J. Oper. Res., vol. 242, no. 3, pp. 710–729, 2015, doi: doi:0.1016/j.ejor.2014.10.008.
- As we will see, the multi-tree method allows us to exploit the separable structure of problem (P).

- Set LB $= -\infty$, UB $= \infty$, and $\varepsilon > 0$.
- For $j = 1, \dots,$ IterMax do :
  - **Step 1.** Solve a MILP relaxation of (P).
  - If the relaxation is infeasible, stop. Otherwise, get new lower bound LB and trial point $v^{(j)}$.
  - **Step 2**. Let UB_j be the optimal value of problem:

$$\begin{aligned} \text{minimize} \quad & \sum_{t=1}^{n_t} c^T x_t \\ \text{subject to} \quad & Ax_t + f(q_t) = 0, \quad \forall t \in \{1, \dots, n_t\} \\ & B_t x_t + C_t q_t \leq e_t - D_t v^{(j)}, \quad \forall t \in \{1, \dots, n_t\} \\ & x_t \in X_t, q_t \in Q_t \quad \forall t \in \{1, \dots, n_t\} \end{aligned} \qquad (\text{UBP}_j)$$

  - If $\text{UB}_j < \text{UB}$, then $\text{UB} = \text{UB}_j$.
  - If $UB - LB \leq \varepsilon$, stop. Otherwise, add constraint $v \neq v^{(j)}$ to the MILP relaxation and go back to Step 1.

Ingredients

- A MILP relaxation of (P).
- A method to solve the nonconvex Problem (UBP$_j$).
- Formulation of constraint $v \neq v^{(j)}$ within the relaxation of (P).

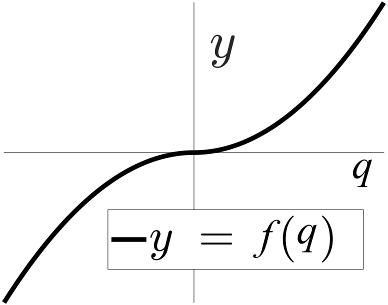
Define a MILP relaxation

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{n_t} c^T x_t \\
\text{subject to} \quad & A x_t + f(q_t) = 0, \quad \forall t \in \{1, \dots, n_t\} \\
& B_t x_t + C_t q_t + D_t v \le e_t, \quad \forall t \in \{1, \dots, n_t\} \\
& x_t \in X_t, q_t \in Q_t \quad \forall t \in \{1, \dots, n_t\} \\
& \sum_{i=1}^{m} v_i = n_v \\
& v \in V \cap \{0, 1\}^m
\end{aligned}
$$

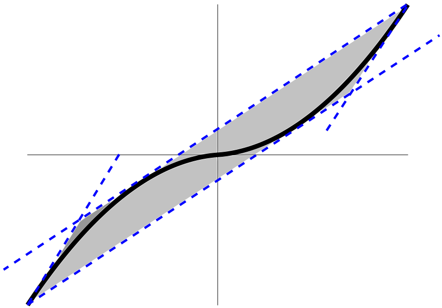
Define a MILP relaxation

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{n_t} c^T x_t \\
\text{subject to} \quad & E_t x_t + R_t q_t \leq r_t, \quad \forall t \in \{1,\dots,n_t\} \\
& B_t x_t + C_t q_t + D_t v \leq e_t, \quad \forall t \in \{1,\dots,n_t\} \\
& x_t \in X_t, q_t \in Q_t \quad \forall t \in \{1,\dots,n_t\} \\
& \sum_{i=1}^{m} v_i = n_v \\
& v \in V \cap \{0,1\}^m
\end{aligned}
\qquad \text{(LBP)}
$$

Define a MILP relaxation



Define a MILP relaxation



Optimization-based bound-tightening

idea: improve the polyhedral relaxation by solving, for each $i = 1, \ldots, n$ and $t = 1, \ldots, n_t$, the following:

$$
\begin{aligned}
\text{minimize/maximize} \quad & q_{it} \\
\text{subject to} \quad & E_t x_t + R_t q_t \leq r_t, \quad \forall t \in \{1, \ldots, n_t\} \\
& B_t x_t + C_t q_t + D_t v \leq e_t, \quad \forall t \in \{1, \ldots, n_t\} \\
& x_t \in X_t, q_t \in Q_t \quad \forall t \in \{1, \ldots, n_t\} \\
& \sum_{i=1}^{m} v_i = n_v \\
& v \in V \cap [0, 1]^m
\end{aligned}
$$

output: A tightened Q_t^{tight} for all $t = 1, \ldots, n_t$

Ingredients

- A MILP relaxation of (P). ✓
- A method to solve the nonconvex Problem (UBP$_j$).
- Formulation of constraint $v \neq v^{(j)}$ within the relaxation of (P).

Solution of UBP

Problem (UBP$_j$) is separable with respect to time index $t \in \{1, \ldots, n_t\}$:

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{n_t} c^T x_t \\
\text{subject to} \quad & Ax_t + f(q_t) = 0, \quad \forall t \in \{1, \ldots, n_t\} \\
& B_t x_t + C_t q_t \leq e_t - D_t v^{(j)}, \quad \forall t \in \{1, \ldots, n_t\} \\
& x_t \in X_t, q_t \in Q_t \quad \forall t \in \{1, \ldots, n_t\}
\end{aligned}
$$

$\Rightarrow$ we can solve n_t different non-convex nonlinear programs:

$$
\begin{aligned}
\text{minimize} \quad & c^T x_t \\
\text{subject to} \quad & Ax_t + f(q_t) = 0 \\
& B_t x_t + C_t q_t \leq e_t - D_t v^{(j)} \\
& x_t \in X_t, q_t \in Q_t
\end{aligned}
\qquad (\text{UBP}_{jt})
$$

Solution of UBP

- Each UBP$_{jt}$ can be solved using an off-the-shelf global optimization solver, computing a δ_{jt}-optimal solution.
- To guarantee that the multi-tree algorithm converges to an ε-optimal solution, we require that $\varepsilon_j = \sum_{t=1}^{n_t} \delta_{jt} \leq \varepsilon$.
- However, solving UBP$_{jt}$ still poses significant computational challenges to off-the-shelf solvers, especially for larger WDNs.
- In practice, we have found that this assumption can be relaxed obtaining good quality solutions of Problem (P).

Solution of UBP

- Assume that, at each iteration j, we compute an un upper bound to the optimal value of (UBP$_j$), denoted by UB$_j$, and the corresponding optimality gap ε_j.
- In this case, the final optimality gap will be

$$\text{UB} - \min_j (\text{UB}_j - \varepsilon_j)$$

  where UB is the best known upper bound.
- Of course, if $\varepsilon_j \leq \varepsilon$ for all j, then the final optimality gap is ε.

Solution based on OBBT

- Implement a non-linear programming solver (e.g. IPOPT) to solve (UBP$_{jt}$) $\rightarrow$ UB$_{jt}$
- For $k = 1, \ldots, \texttt{Kmax}$:
  - Apply OBBT to variables q_t.
  - Compute optimality gap by solving:

$$\begin{aligned} \text{minimize} \quad & c^T x_t \\ \text{subject to} \quad & E_t x_t + R_t q_t \leq r_t \\ & B_t x_t + C_t q_t \leq e_t - D_t v^{(j)} \\ & x_t \in X_t, q_t \in Q_t \end{aligned}$$

  - When the optimality gaps reduction rate is smaller than 1%, stop.

Ingredients

- A MILP relaxation of (P) ✓.
- A method to solve the nonconvex Problem (UBP$_j$) .✓
- Formulation of constraint $v \neq v^{(j)}$ within the relaxation of (P).

Update lower bounding problem

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{n_t} c^T x_t \\
\text{subject to} \quad & E_t x_t + R_t q_t \leq r_t, \quad \forall t \in \{1, \ldots, n_t\} \\
& B_t x_t + C_t q_t + D_t v \leq e_t, \quad \forall t \in \{1, \ldots, n_t\} \\
& x_t \in X_t, q_t \in Q_t \quad \forall t \in \{1, \ldots, n_t\} \\
& \sum_{i=1}^{m} v_i = n_v \\
& \sum_{i=1}^{m} v_i^{(j)} v_i \leq n_v - 1 \\
& v \in V \cap \{0, 1\}^m
\end{aligned}
$$

Ingredients

- A MILP relaxation of (P) .✓
- A method to solve the nonconvex Problem (UBP$_j$) .✓
- Formulation of constraint $v \neq v^{(j)}$ within the relaxation of (P).✓

Numerical Experiments

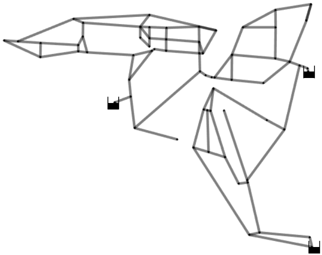


Figure: Benchmarking network of Pescara.

- 70 nodes, 98 pipes, 24 time steps
- 5 problems instances with $n_v \in \{1, \ldots, 5\}$.
- 6312 continuous variables.
- 196 binary variables.
- 2352 non-convex terms.

All experiments are carried out in Julia 1.7.2, where optimization solvers are called through JuMP 1.0. All nonlinear programs (UBP) are solved with IPOPT, while MILPs and LPs are solved by GUROBI.

Results

Table: Multi-tree algorithm with tolerance = 5% and maximum number of iterations = 80.

n_v	UB	Gap (%)	Outer iterations	CPU time (s)
1	43.57	3.89	8	1003
2	34.43	4.02	22	3195
3	28.72	4.64	39	5287
4	24.16	4.96	47	5993
5	23.32	5.65	80	10127

Table: Best solution computed by BARON/SCIP/OCTERACT

n_v	UB	Gap (%)	CPU time
1	43.57	53.10	21600
2	34.49	53.50	21600
3	28.24	33.39	21600
4	∞	∞	21600
5	∞	∞	21600

Conclusions

- The problem of optimal design-for-control of water networks is difficult to solve because it can result in challanging MINLPs even for small case studies.
- Multi-tree decomposition methods offer an effective strategy to exploit problem structure and compute feasible solutions with small optimality gaps.
- There is more work to do! The global optimization community can contribute in many ways to solve these critical societal problems:
  - Strengthen the MILP relaxation deriving optimality cuts from solutions of UBP.
  - Implement standard Benders decomposition to solve the relaxed MILP when large operational networks are considered.
  - The computational effort required by OBBT grows rapidly with network size, better methods to globally optimize UBP?

Thank you!

Acknowledgements
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For further information

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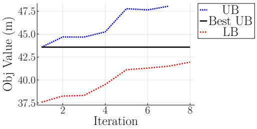
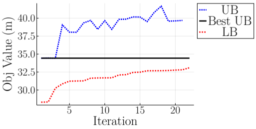
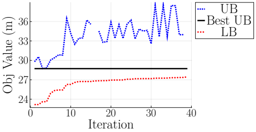
Appendix



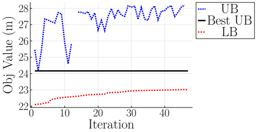
(a) $n_v = 1$



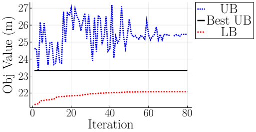
(b) $n_v = 2$



(c) $n_v = 3$



(d) $n_v = 4$



(e) $n_v = 5$